# DISCUSSION ON THE RULES FOR COMBINATION OF ACTIONS IN EN 1990 "BASIS OF STRUCTURAL DESIGN"

## Foreword

EN1990: 2002 *Basis for structural design* provides for countries to choose between three load combination expressions, without giving criteria for making choices.

CEMBUREAU/BIBM/ERMCO commissioned BRE to review an earlier study prepared by SAKO, the Joint Nordic Group for Structural Matters, on behalf of NKB and INSTA-B. This study had been carried out with the objective of "comparing the level of consistency of safety for various ratios between the permanent and variable actions by considering the three principal structural materials: concrete, steel and glulam timber". The Nordic report demonstrated that the three load combination expressions give quite differing levels of reliability for different ratios of variable load to total load.

The BRE study confirmed that all three load combination expressions, when used with appropriate partial and combination factors, should still achieve current target levels of reliability.

Opportunity has been taken to clarify some misunderstandings, which have appeared in some communications.

CEMBUREAU Project Group PG 2.5 *Eurocodes*, comprising members from CEMBUREAU, BIBM and ERMCO, welcomes the work done by CERIB and supports their conclusions.

Dr John Moore
*Chairman of CEMBUREAU PG 2.5 "Eurocodes"*

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# Discussion on the rules for combination of actions in EN 1990 "Basis of Structural Design"

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## 1. Introduction

**Several documents have been published on the rules for the combination of actions in EN 1990 and the prior ENV version (see references). It is generally accepted that the use of a single such as equation 6.10 of EN 1990 will lead to variable levels of safety across the range of loads. Also it is generally accepted that the safety is higher in structures when permanent or self-weight loads are dominant. Therefore the use of equations 6.10 a and 6.10 b is recommended for obtaining a consistent level of safety across the full range of loads. Opportunity has been taken to comment on misunderstandings in some published papers.**

## 2. Requirements of the verification format given in EN 1990

(1) Verification of safety by the partial factor method requires, for standardisation purposes, that the two terms of the verification equation ($E_d$: design value of effect of actions and $R_d$: design value of the resistance) are independent from each other. Consequently there is a need for a uniform level of safety given by the two terms (Ed) and ($R_d$). In other words, the probability of exceedance of the design value of effect of actions ($E_d$) given by the load combination expression should be as far as possible constant whatever the values of the individual actions; similarly the probability that the actual resistance of an element or a structure is less than the design value ($R_d$) should be kept constant whatever the design.

## 3. Representative values for actions and statistical distributions

(1) The different actions to be considered in design are (see Background Documentation Eurocode 1[2], [4]):

- the permanent actions (G). The statistical distribution of G may be assumed to be Gaussian and the characteristic value $G_k$ is taken equal to the mean value (50 % fractile). It should be noted that the variability of G depends on its origin. For dead loads, like partitions and non-structural elements, a coefficient of variation of 10 % may be assumed. For self-weight of a structure the coefficient is seldom higher than 5 %, and can be reduced to 2 % [5]. Contrarily, the coefficient of variation of certain permanent loads like loads caused by settlements and earth weight can be significantly higher.
- the variable actions (Q), including climatic loads and imposed loads. The statistical distribution of Q is assumed to be well-described by a Gumbel distribution. For climatic

loads (wind and snow), the characteristic value is defined by its probability (p) to be exceeded during a reference period (R). The probability (p) generally adopted is equal to 0.02, calculated over a one-year period (R=1). These two parameters can be replaced by the return period (T) which corresponds to the mean duration between consecutive occurrences of $Q_k$ being exceeded:

$$T = -\frac{R}{\ln(1-p)} \qquad (1)$$

For imposed loads, the characteristic value is generally set to the 95 percentile for a reference period of 50 years. The following figure from [2] illustrates the different representative values of a variable action.

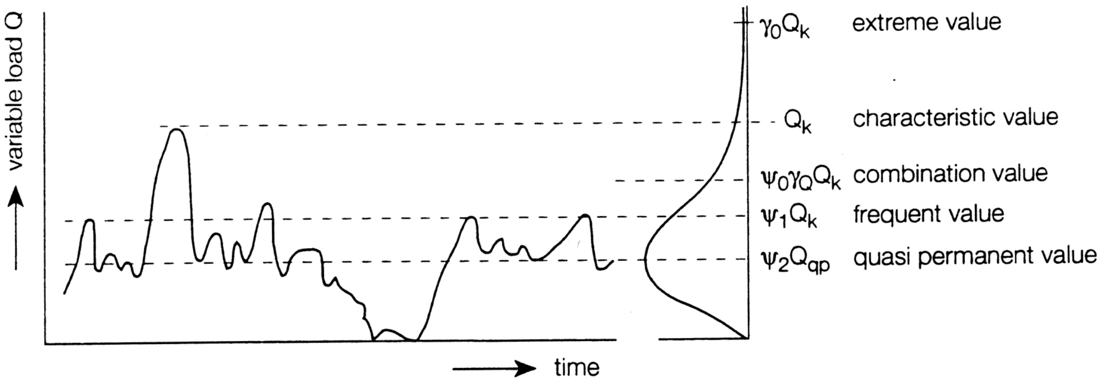


Figure 1: representative values of Q, from [2].

(2) The assumption adopted in [1], "Commentary to prEN 1990" for the numerical comparison of the combination rules is rather different:

For variable action, it is assumed that the characteristic values are calibrated with the partial safety factor $\gamma_Q = 1{,}5$. Adopting, as stated, a reference period of 50 years, this leads to a non-constant return period for the variable load, depending on the coefficient of variation of the variable load itself. The following assumption for $k_n$, an indicator defining the characteristic value used for design, is used:

$$k_n = \frac{2{,}97V_Q - 0{,}4}{\gamma_Q V_Q} \text{ for } \gamma_{Sd} = 1{,}10 \text{ and } Q_k = \overline{Q}(1 + k_n V_Q)$$

→ if $V_Q = 0{,}2$ then $k_n = 0{,}646$ and $Q_k = 1.13\overline{Q}$ which corresponds to a 74%-fractile (p=0.26) and leads to the return period T=167 years.

→ if $V_Q = 0{,}4$ then $k_n = 1{,}313$ and $Q_k = 1.52\overline{Q}$ which corresponds to a 90.5%-fractile (p=0.1) and leads to the return period T=500 years.

EN 1990 (section 4.1.2) [3] recommends for characteristic values of climatic actions p=0.02 and T=50 years. Adopting implicitly higher fractile values or higher return period values means that the corresponding required partial factor will be less than the one implied by the assumptions of the code.

# 4. Evaluation of the combination rules

## *4.1 Partial factors for dominant actions*

(1) The general format for the calculation of a design value using the simplified FORM method is as follows (See “Basis of Structural Design”, annex C [3]):

$X_d = X\,(1 - \alpha\beta V)$ for a Normal distribution
$X_d = u - 1/a\,\ln(-\ln(\Phi(\alpha\beta))$ for a Gumbel distribution
Where :
X : mean value of the variable x,
$\alpha$ : sensitivity factor,
$\beta$ : reliability index,
V : coefficient of variation of the variable x,
u = X – 0,577 / a,

$a = \frac{\pi}{\sqrt{6}XV}$.

The sensitivity factor models the relative influence of each variable in the limit state function. The recommended values are:
$\alpha_E = -0.7$ for actions and $\alpha_R = 0.8$ for resistance.

(2) As mentioned in [3], the partial factors for actions can be obtained by dividing the design value by the characteristic value. It should be noted that an additional factor should be included to take into account model uncertainty on actions and action effects ( $\gamma_{Sd}$ ):

$$\gamma_E = \frac{\mathrm{E_d}}{\mathrm{E_k}}\gamma_{Sd} \qquad (2)$$

The application of these equations, with a model uncertainty factor equal to 1,05 and a sensitivity factor equal to – 0,7 will lead to the following required partial factors with respect to a target reliability index of 3,8 and a return period of 50 years for variable loads [2]:

Table 1: permanent loads

| Load | $V_G$ | $\gamma_G$ |
|---|---|---|
| Dead load | 0,10 | 1,33 |
| Self weight | 0,05 | 1,19 |

Table 2: variable loads

| Load | $V_Q$ (for T = 50 years) | $\gamma_Q$ |
|---|---|---|
| Wind | 0,20 | 2,04 |
| Snow (land climate) | 0,15 | 1,77 |
| Snow (sea climate) | 0,30 | 2,67 |
| Imposed (20m²) | 0,30 | 1,50 |

As it can be easily deduced, adopting a unique $\gamma_G$ and $\gamma_Q$ value, for simplification, will lead to a different safety level with respect to the type of loading. This is clearly stated in [2]. Some hidden conservative assumptions compensate partly, especially for wind.

## *4.2 Global Safety Factors and combination rules*

(1) When two or more independent actions are considered in the model, the combination coefficient ($\psi_0$) takes into account that the probability of two or more actions exceeding their characteristic values at the same time is smaller than the probability that one single action exceeds its characteristic value [4]. The alternative set of expressions 6.10 a) and 6.10 b) is derived from this observation [2]. The recommended values for $\psi_0$ [I suggest the convention is no brackets for a symbol on its own but brackets when the symbol follows a descriptiuon of it. I leave you to make further changes in line with this convention] are generally 0,7 for imposed load, 0,6 for wind load and for permanent loads the reduction factor ($\xi$) has the same role (recommended value is 0,85).

(2) As mentioned above, "Basis of structural design" annex C [3] provides a simplified FORM method that can be used for the determination of partial factors. The general case, where several independent actions are present, is treated by differentiating the sensitivity factors for the leading variable and for the accompanying actions. For this latter, the sensitivity factor should be multiplied by 0,4: $\alpha_E = -0{,}4\,x\,0{,}7 = -0{,}28$.

(3) in [1], "Commentary to prEN 1990", the notion of global safety factor is introduced as follow:
the "required" global safety factor, given the scatter of the different loads is given by the above expression,

$$\gamma_{\mathrm{G+Q}\,req} = \frac{\mathrm{E_d}}{(\mathrm{G_k}+\mathrm{Q_k})} = \frac{\gamma_{\mathrm{Sd}}\left[\overline{G}(1+\tilde{\alpha}_G\alpha_E\beta V_G)+\overline{Q}(1+\tilde{\alpha}_Q\alpha_E\beta V_Q)\right]}{G_k+Q_k} \qquad (3)$$

and the "resulting" global safety factor, given by the application of expressions 6.10 a) and 6.10 b):

$$\gamma_{\mathrm{G+Q}} = \frac{\mathrm{E_d}}{(\mathrm{G_k}+\mathrm{Q_k})} = \frac{Max(\xi\gamma_{\mathrm{G}}G_k+\gamma_Q Q_k\ ;\ \gamma_{\mathrm{G}}G_k+\psi_0\gamma_Q Q_k)}{G_k+Q_k} \qquad (4)$$

with:
$$\left|\begin{array}{ll} G_k = & \overline{G} \\ Q_k = & \overline{Q}(1+k_n V_Q) \\ \tilde{\alpha}_G = & \dfrac{V_G\overline{G}}{\sqrt{(V_G\overline{G})^2+(V_Q\overline{Q})^2}} \\ \tilde{\alpha}_Q = & \sqrt{1-\tilde{\alpha}_G^2} \end{array}\right.$$

In their analysis, the authors had chosen to refine the two equations (3) and (4) by considering ($\tilde{\alpha}_i$) and ($\psi_0$) as functions of ($V_Q$) and ($\chi = \dfrac{Q_k}{Q_k+G_k}$). These developments are really interesting but, on the other hand, the required global load safety factor is also influenced by the ($\alpha_E$) coefficient for a given structural material. It is clear that ($\alpha_E$) will increase when variable load with high variation become dominant and this will increase accordingly the required global load safety factor. The effect is the opposite when a load with small variation, like self-weight, becomes dominant. The value ($\alpha_E = -0{,}7$) is a safe sided choice corresponding to dominant variable actions. The "required" calculated coefficient will hence be higher than necessary for dominant permanent loads. Although more complex calculations are needed, the use of reliability index ($\beta$) allows all aspects of the problem to be taken into account and is recommended for comparison of the safety provided by different combinations. Extensive calibrations have been achieved in [6] using this later method.

(5) The following figure represents the evolution of ($\gamma_{\mathrm{G+Q}}$ )$_{\mathrm{req}}$ versus ($\chi$), using formula (3), for $\gamma_{Sd} = 1.05$:

i) as it had been calculated in [1],

ii) then, by considering a constant return period for variable loads T=50 years (see table 2)

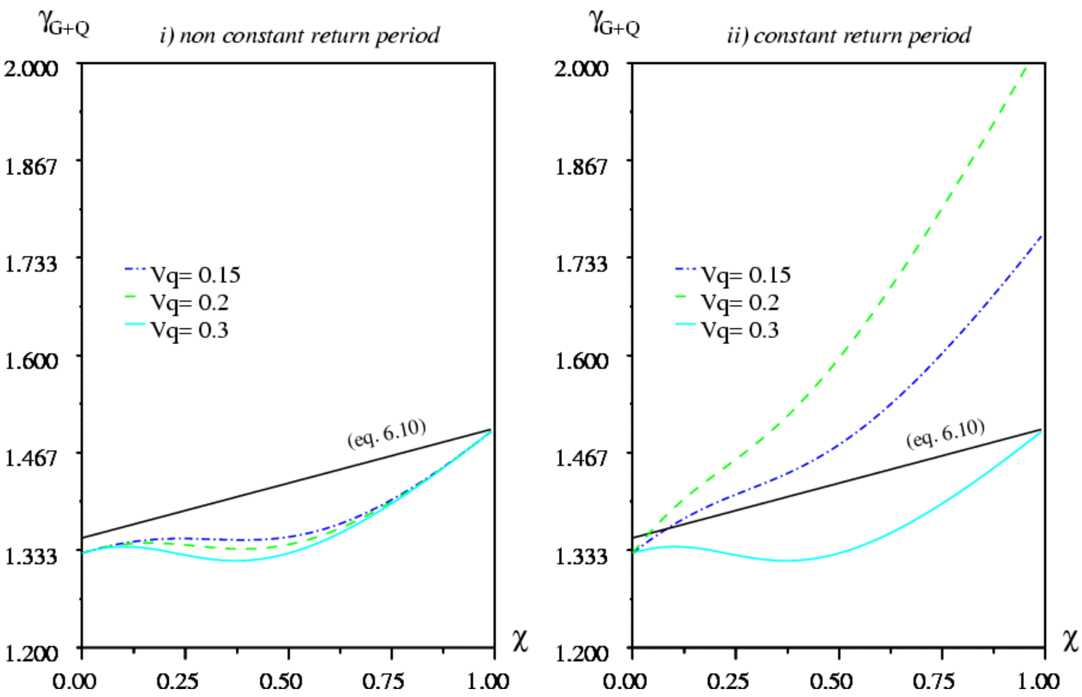


Figure 2: ($\gamma_{\mathrm{G+Q}}$ )$_{\mathrm{req}}$ evolution versus ($\chi$)

(6) From figure 2, it can be seen that introducing the usual assumptions for load models leads to higher levels of "required" load safety factor than calculated in [1] for dominant variable load (figure 2, right).

## 5. Safety comparison for the different load combination expressions given by EN1990

(1) In the following, the probabilities ($P_i$), that the total load will exceed the design load have been calculated using the Monte-Carlo method:

$$P_i = P(\mathrm{g} + \mathrm{q} > C_i) \qquad (5)$$

(2) Calculations have been made for different load combination expressions, given by the EN-1990:

$$C_1 = \gamma_g G_k \quad + \quad \gamma_q Q_k \qquad (eq.6.10)$$
$$C_2 = \gamma_g G_k \quad + \quad 0{,}7 \times \gamma_q Q_k \quad (eq.6.10a)$$
$$C_3 = 0{,}85 \times \gamma_g G_k \quad + \quad \gamma_q Q_k \quad (eq.6.10b)$$

with $\gamma_g = \dfrac{\gamma_G}{\gamma_{Sd}} = \dfrac{1.35}{1.05} = 1.28$,

and $\gamma_q = \dfrac{\gamma_Q}{\gamma_{Sd}} = \dfrac{1.5}{1.05} = 1.43$.

(3) According to tables 1 and 2, Normal and Gumbel distributions had been assumed respectively for permanent and variable loads.

(4) Each value of (P) had been calculated with 1 million simulations, at least.

(5) The below figure shows probabilities calculated from expression 6.10 for 3 different types of variable load. This figure confirms that the probability (P) increases in the interval $0.5 \le \chi \le 1$ when increasing the ($V_Q$) coefficient (for permanent load, $V_G$=10 %, $G_k$ = 50 % fractile)

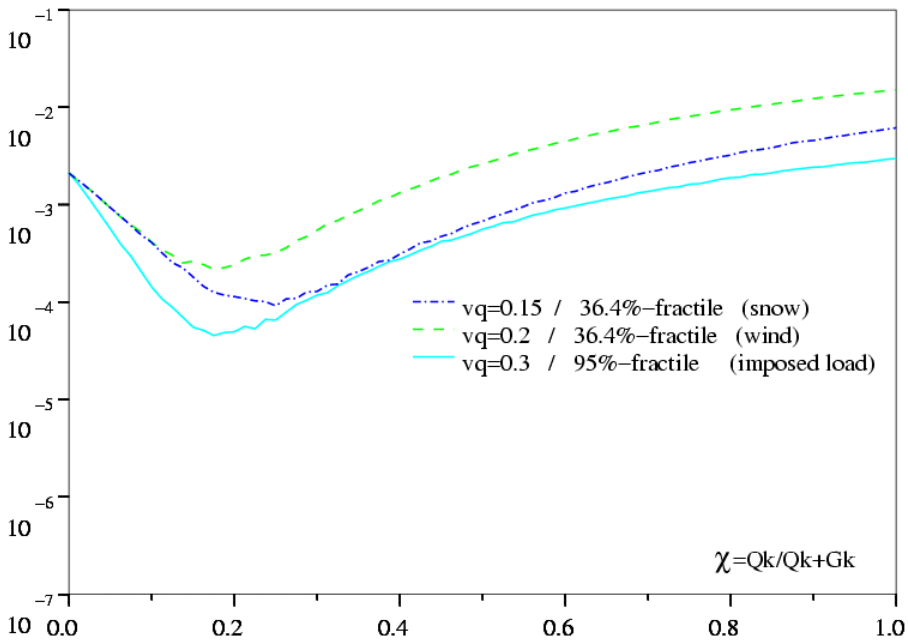


Figure 3: Exceedance probabilities, calculated from expression 6.10

(6) The next figure shows the ($P_i$) evolution versus ($\chi$) for the three load combinations ($C_i$).

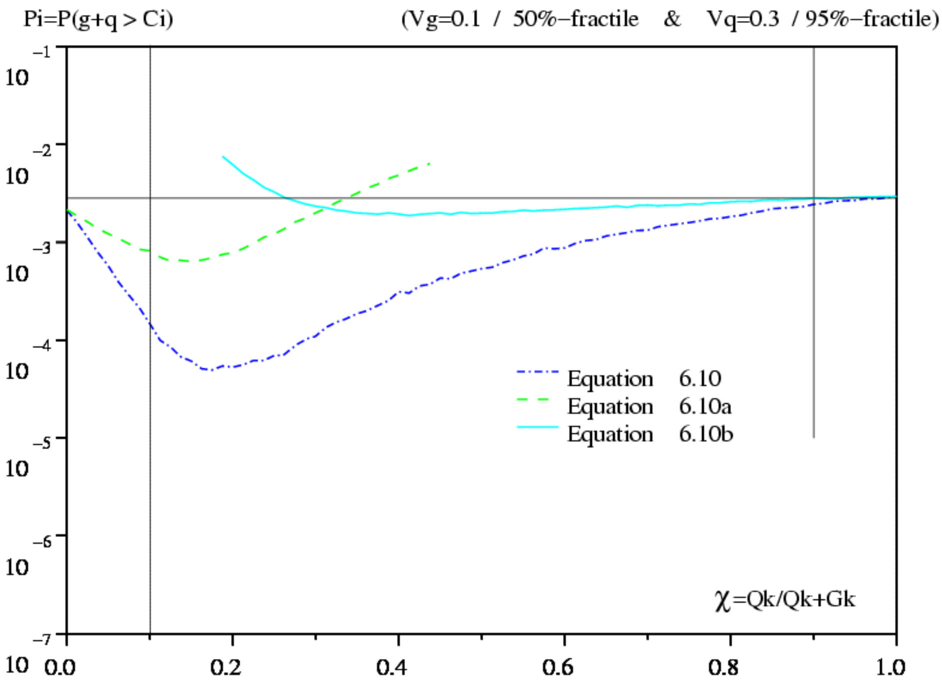


Figure 4: Exceedance probabilities, calculated from expressions 6.10 and 6.10a) and b)

# 6. Conclusions

From the above investigations it can be said that:

The calculations achieved in [1], based on simplified models and on the strong assumption that a partial factor for variable load equal to 1,5 gives a constant reliability whatever the variable load, are not appropriate.

If the usual assumptions for variable and permanent loads are adopted, according to reference documents [2], [3], [4], the variation of the required global safety factor for loads changes and the value becomes much higher, especially when climatic loads dominate.

In order to compare the different combination expressions, the probability of exceedance of design load values has been investigated, adopting the accepted assumptions for load models. From these calculations, it can be seen that the use of expressions 6.10 a and 6.10 b leads to an exceedance probability not greater than the maximum probability given by expression 6.10 for the possible range of load ratios.

For a complete comparison of safety provided by the different sets of expressions in EN 1990, the material variability versus the load variability should be considered. Reference is made to [6], where extensive simulations have been done.

## References

[1] *Commentary to prEN 1990 – Basis of Design*. J. Brozzeti, G. Sedlacek, O. Kraus, [Private communication]
[2] Background documentation, Eurocode 1 (ENV 1991), Part 1: Basis of design, JCSS, March 1996
[3] EN 1990 (2002) “Basis of structural design”
[4] *Survey and background of the semi-probabilistic design method for concrete structures according to Eurocodes EC1 and EC2.* L. Taerwe - University of Ghent, Dept. of Structural Engineering, Ghent, Belgium
[5] Precast concrete safety factors – European research, Contract SMT4 CT98 2276, July 2002
[6] "Safety of Structures", An independent technical expert review of partial factors for actions and load combinations in EN 1990 "Basis of Structural Design", BRE, 2003